Performance of Decision-Aided Maximum-Likelihood Carrier Phase Estimation with Frequency Offset

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Abstract: Adaptive decision-aided maximum-likelihood (DA ML) carrier phase estimation (PE) is shown to be more robust to frequency offset, with better frequency-offset and laser-linewidth tolerance compared to conventional DA ML carrier PE.
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1. Introduction

A major challenge in coherent optical phase modulated systems is the recovery of the carrier phase for coherent detection. An optical phase locked loop (PLL) can be implemented to track the carrier phase, but its operation at optical wavelengths in combination with distributed feedback lasers is difficult to implement in real life as the product of the laser linewidth and loop delay can be large, driving the PLL to instability [1]. Digital signal processing (DSP) algorithms for optical coherent detection are now increasingly implemented to perform carrier phase estimation (PE). An *M*th power scheme was proposed in [2] to estimate the phase reference by raising the received *M*-ary PSK signal to its *M*th power. However, this scheme requires nonlinear operations such as *M*th power and $arctan(\cdot)$. Moreover, *M*th power scheme introduces a $\pm 2\pi/M$ phase ambiguity thus necessitating phase unwrapping and is prone to cycle slipping which is a highly nonlinear phenomenon [3].

Hence, a computationally linear, decision-aided, maximum-likelihood (DA ML) PE technique with no phase ambiguity and no phase unwrapping was proposed in [4]. DA ML PE has bit error rate (BER) performance comparable to the *M*th power scheme in linear phase noise systems but outperforms the *M*th power scheme in nonlinear phase noise dominant systems [5]. Both *M*th power and DA ML suffer from block length effect (BLE) which refers to the effect of the selected memory length on the accuracy of the PE [6]. To overcome the BLE in DA ML, a comparatively superior, first-order, adaptive DA ML PE was developed in [6] which is independent of BLE.

Impeding the successful carrier phase recovery by DSP based PE algorithms is the presence of large frequency offset, Δf between the transmitter and local oscillator (LO) lasers [7]. In this paper, we demonstrate the superiority of adaptive DA ML in carrier phase estimation while adapting better to frequency offsets compared to the conventional DA ML, with no additional frequency offset estimators used. The laser linewidth tolerance for a given frequency offset and *vice versa* is investigated showing the comparatively higher robustness of adaptive DA ML.

2. DA ML and adaptive DA ML model

The received signal in digital coherent receiver can be represented by $r(k) = m(k)\exp\left[j\left(\omega k + \theta(k)\right)\right] + n(k)$, where k denotes kth symbol interval $[kT, (k+1)T\ )$ (T is the symbol duration), $m(k) \in \left\{m_i = \sqrt{E_s}\exp[j\phi_i(k)]\ \right\}$ is the data symbol, E_s is the M-*ary* PSK symbol energy, and $\omega = 2\pi(\Delta f)$ is the frequency offset. Here, $\theta(k)$ is the phase noise introduced by laser linewidth, which is modeled as a Weiner process, $\theta(k) = \theta(k-1) + \nu(k)$. The sequence $\{\nu(k)\}$ is a set of independent, identically distributed Gaussian random variables, each with mean zero and variance $\sigma_p^2 = 2\pi(\Delta\nu)T$ where $\Delta\nu$ is the combined linewidth of the transmitter and LO lasers. The sequence $\{n(k)\}$ is the complex, additive, white, Gaussian noise (AWGN), each with $\mathrm{E}[n(\mathrm{k})] = 0$ and $\mathrm{E}[|n(\mathrm{k})|^2] = \sigma_n^2$. The signal to noise ratio (SNR) per symbol is defined as $\gamma_s = E_s/\sigma_n^2$ whereas the SNR per bit is given by $\gamma_b = \gamma_s/\log_2 M$.

In DA ML PE, the maximum likelihood phase estimate $\hat{\theta}(k)$ at time $t = kT$ is computed using the immediate past L received signals, where L is the memory length. A complex reference phasor (RP) $V(k+1)$ is formed as $\sum_{l=k}^{k-L+1} r(l)/\hat{m}(l)$ where $\hat{m}(l)$ is receiver's decision on the lth received symbol. Each term $r(l)/\hat{m}(l)$ contains the residual phase contributed by laser phase noise, frequency offset and the AWGN component. The decision statistic is then given by $\hat{m}(k) = \arg\max_i \mathrm{Re}[r(k)V^*(k)m_i^*]$. Superscript * denotes complex conjugation. In the first order adaptive DA ML, the previous RP $V(k)$ and the current input $r(k)/\hat{m}(k)$ are weighted by α and $1-\alpha$ respectively to form the next RP, $V(k+1) = \alpha(k)V(k) + \left(1-\alpha(k)\right)r(k)/\hat{m}(k)$. The filter gain $\alpha(k)$ is chosen automatically at each time k based on all past observations to minimize the conditional risk function $R(k) = \mathrm{E}\left[\sum_{l=1}^{k} |r(l) - V(l)\hat{m}(l)|^2 \middle| \{r(l)\}_{l=1}^{k}\right]$ [6]. The decision statistic is the same as that in DA ML. Both DA ML and adaptive DA ML require a preamble sequence to start up the RP estimators and operate in decision-directed mode subsequently.

3. Results and discussion

Monte Carlo simulations were performed to measure the impact of varying frequency offset on the BER performance of DA ML and adaptive DA ML for a given combined transmitter and LO laser linewidth of 1.27 MHz in a 40Gbit/s, single polarization, quadrature phase shift keying (QPSK) system. Symbol timing is assumed to be known and the symbols are differentially encoded to prevent error propagation due to decision feedback errors. An initial known set of 100 symbols is used as the preamble sequence. To ensure a simple and fair frequency offset effect comparison between DA ML and adaptive DA ML, the memory length of DA ML is set to the optimum length given by $L_{opt} = \lfloor 0.25\sqrt{1+24\sigma_n^2/\sigma_p^2} - 0.75 \rfloor$ at every γ_b [6]. Note that the L_{opt} was derived to yield the lowest BER and thus optimality only for a zero frequency offset case.

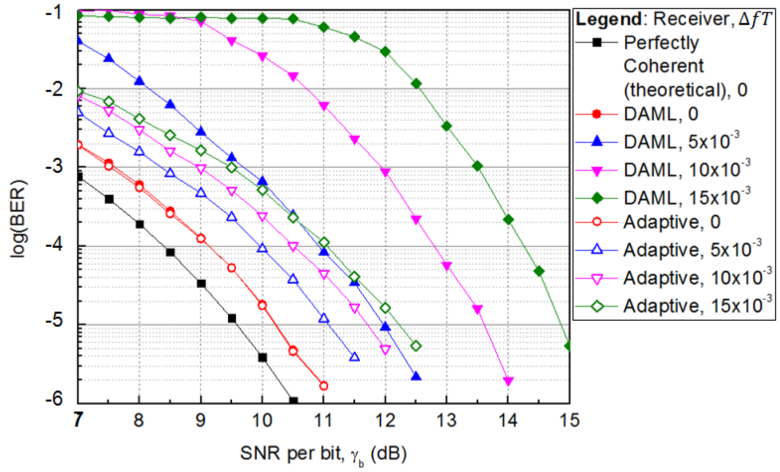

Fig. 1. Performance comparison of DA ML and adaptive DA ML for fixed combined laser linewidth of 1.27 MHz at 40 Gbit/s. Legend key indicates the type of receiver used and its corresponding frequency offset per symbol rate value, ΔfT used.

Shown in Fig. 1, at zero frequency offset, the DA ML BER curve overlays that of adaptive DA ML. This is because, optimizing DA ML's memory length with respect to phase noise yields the same BER performance as adaptive DA ML for any given laser linewidth at zero frequency offset. As a performance measure, γ_b penalty at $\text{BER} = 10^{-5}$ is measured in reference to the γ_b of a perfectly coherent (theoretical) phase noise free and zero frequency offset system with bit error probability $= \frac{1}{2}erfc\left(\sqrt{\gamma_b}\right)$. At a frequency offset per symbol rate of $\Delta fT = 5 \times 10^{-3}$, DA ML incurs a γ_b penalty of 2.38 dB whereas adaptive DA ML incurs a lesser γ_b penalty of 1.48 dB, resulting in a 0.9 dB gain by adaptive DA ML over the conventional DA ML. The gain of adaptive DA ML over the conventional DA ML increases, with increasing frequency offset, to 1.9 dB at frequency offset per symbol rate of 0.01 (1% symbol rate).

The phasor component of $r(k)/\hat{m}(k)$, which is the input to form the RP, can be represented as $\exp\left[j\left(\omega k + \theta(k) + \epsilon(k)\right)\right]$ where $\epsilon(k)$ denotes the phase contribution by AWGN component. As seen from the phasor equation, the current input $r(k)/\hat{m}(k)$ will be phase mismatched and thus less correlated with the input term l symbol periods ago (i.e. $r(k-l)/\hat{m}(k-l)$) due to the constant phase rotation term ω. Therefore, at higher frequency offsets, the past terms will decorrelate faster with the current input and thus their contribution to formation of RP needs to be weighted down to obtain a better phase tracking. In conventional DA ML, the past L inputs are equally weighted without consideration for their importance thereby obtaining a poor phase estimate. On the other hand, the adaptive DA ML weighs down past input terms with varying degree of decay as determined by the filter gain $\alpha(k)$ (see RP formation equation given above for adaptive DA ML) which is automatically chosen, by minimizing the risk function. Therefore, adaptive DA ML yields a better phase estimate.

In Fig. 2 the γ_b penalty as a function of frequency offset per symbol rate using DA ML and adaptive DA ML for various given laser linewidths is plotted. With DA ML, the γ_b penalty increases more rapidly for lower laser linewidths per symbol rate ($\Delta\nu T$). This is because the DA ML memory length, as determined by L_{opt}, increases with decreasing σ_p^2 and thus a larger sequence of $\{r(l)/\hat{m}(l)\}$ with less correlated terms are averaged resulting in poorer phase estimate. The γ_b penalty increases at the same rate for various laser linewidths when adaptive DA ML is used implying that it is less sensitive to varying laser linewidths. This is due to its ability to adaptively prioritize the input terms in the input sequence during RP computation. The limiting frequency offset at a given laser linewidth per symbol rate of $\Delta\nu T = 10^{-5}$ (corresponding to a laser linewidth of 200 kHz in our case) which leads to

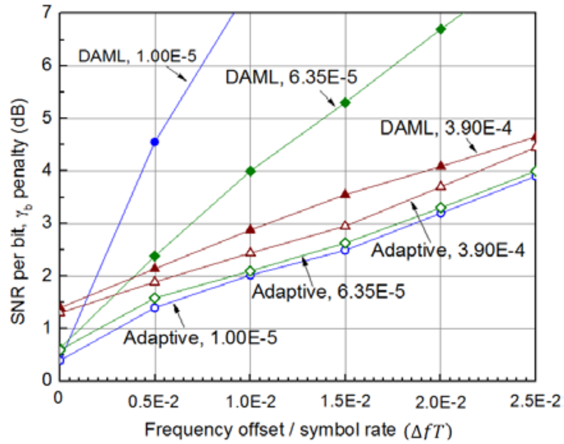

Fig. 2. Comparison of DA ML and adaptive DA ML for fixed laser linewidth as a function of frequency offset/symbol rate. Labels indicate receiver type and laser linewidth/symbol rate, $\Delta\nu T$ used.

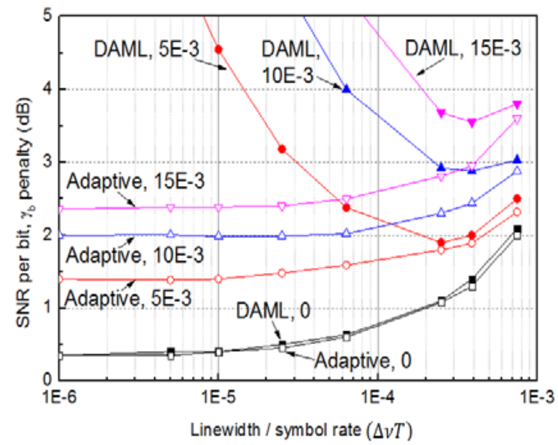

Fig. 3. Comparison of DA ML and adaptive DA ML for fixed frequency offset as a function of laser linewidth/symbol rate. Labels indicate receiver type and frequency offset/symbol rate, ΔfT used.

a 2 dB γ_b penalty for adaptive DA ML, is 0.16 GHz (0.8% symbol rate) higher than that of conventional DA ML. This shows adaptive DA ML's improved tolerance to increased frequency offset.

To investigate the laser linewidth tolerance at a given frequency offset, the γ_b penalty is plotted as a function of linewidth per symbol rate in Fig. 3. In DA ML, the γ_b penalty decays faster initially for higher given frequency offset. Since DA ML PE's memory length, L_{opt} is recomputed for every laser linewidth, DA ML effectively uses smaller memory length at larger laser linewidths. Smaller memory length benefits PE by averaging over smaller set of inputs which are more correlated resulting in better phase estimate and a decrease of γ_b penalty. For adaptive DA ML with a given frequency offset, the γ_b penalty remains fairly constant for low linewidth per symbol rate ($\Delta\nu T < 3 \times 10^{-4}$) because adaptive DA ML is able to adjust its effective length over which the averaging is performed to compute the RP. Laser linewidth per symbol rate values lower than 6.35×10^{-5} (corresponding to a linewidth of 1.27 MHz in our case) produces smaller $\gamma_b$penalty in adaptive DA ML in comparison with DA ML for the same frequency offset. Therefore adaptive DA ML is more laser linewidth tolerant in the presence of frequency offset.

It is interesting to note that the γ_b penalty of both DA ML and adaptive DA ML increases at higher linewidth per symbol rate, irrespective of given frequency offsets. This is due to DA ML's inherent laser linewidth tolerance limitation to phase noise in the absence of frequency offset as reported in [8]. This result is corroborated in Fig. 3 by the increase of γ_b penalty for zero frequency offset case using either DA ML or adaptive DA ML.

4. Summary

Adaptive DA ML phase estimation for carrier phase tracking is shown to be more robust compared to DA ML in the presence of frequency offset. To compute the optimum memory length for DA ML in the presence of frequency offset, *a priori* knowledge of phase noise variance, σ_p^2, AWGN variance, σ_n^2, and the frequency offset Δf is required which may be impractical in real life especially in a reconfigurable optical system. In stark contrast, adaptive DA ML is able to adjust its optimum effective averaging length through the filter gain $\alpha(k)$, which is automatically computed with no *a priori* knowledge required.

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